Note on Professor Brown's Note. By E. J. Stone.

Professor Brown is quite right in saying that the terms

$$\sin(g+l)\left[+\tfrac{3}{8}\gamma^7\right] \text{ and } \sin(3g+3l)\left[-\tfrac{3}{8}\gamma^7\right]$$

which appear on p. 113 in my paper, "Expressions for the Elliptic Coordinates of a Moving Point to the Seventh Order of Small Quantities," are incorrectly printed.

The first is simply an error of sign. The second has arisen from the coefficient found for $\sin(g+l)$, $-\frac{3}{8}\gamma^7$, having been by inadvertence copied by me, when preparing the copy for press, into the argument for $\sin(3g+3l)$, instead of $-\frac{1}{8}\gamma^7$.

The coefficients of the terms $\sin(5g+5l)$ and $\sin(7g+7l)$, which involve γ^7, and were found conjointly with those above-mentioned, are correctly printed.

Radcliffe Observatory, Oxford:
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On the Relative Efficiency of a Reflector and of Portrait Lenses for the Delineation of Celestial Objects. By Isaac Roberts, D.Sc., F.R.S.

It has often been asserted that portrait lenses have, by reason of their short foci and relative apertures, greater photographic power than instruments constructed on other models, such as a reflector; but the assertion had not been confirmed, on any occasion, within my experience in the past. I therefore considered it desirable to submit the question to the test of some crucial experiments; the results of which shall now be laid before the Society.

The experiments were conducted in the manner following.

Two typical portrait lenses were purchased, of the best quality that could be obtained, and were affixed with their cameras firmly to the tube of the 20-inch reflector. The arrangements were such that three photographs of the same objects, by three different instruments, could be taken simultaneously under precisely and in every respect similar conditions.

The photo-plates were of the same degree of sensitiveness, the durations of the exposures were quite equal, and the development of the negatives was similar in all cases.

The photo-negatives thus obtained were examined; the stars upon equal and coincident areas on each plate were counted, including the faintest stars that could, by aid of a magnifier, be recognised as true star-images. The nebulosity also was compared on each corresponding plate, together with its extent, area covered, density, details, and every distinctive feature noted.

A portrait lens of the latest pattern, and of the most rapid combination, was obtained from Dallmeyer and Co. The aperture is $3\frac{1}{2}$ inches, and stellar focus 9·56 inches. The ratio of aperture to focus is therefore 1 to 2·74, and it covers, without much distortion, a photo-field of about 11 degrees in diameter.

Messrs. Cooke and Sons made specially for me a Taylor's triplet combination lens of 5 inches aperture and 19·22 inches stellar focus. The ratio of aperture to focus is therefore 1 to 3·84, and for these experiments it was used as 1 to 4·8. It covers a photo-field, without much distortion, of about 15 degrees in diameter. My experience of this lens is most satisfactory, and I think it will be difficult to improve upon it.

The reflector has an aperture of 20 inches, focus 98 inches, ratio of aperture to focus 1 to 4·9, and the photo-field is about $2\frac{1}{2}$ degrees in diameter.

The two lenses referred to were affixed, as already stated, upon the tube of the 20-inch reflector, so that a photograph with the $3\frac{1}{2}$-inch, the 5-inch, and the 20-inch instruments could be taken simultaneously, and under all the conditions of equality essential to a full and fair trial.

The results of the experiments may now be given, but in order to obviate too much repetition, I will give only three typical examples and illustrations, for they will be representative of the whole series.

We will first project on the screen a photograph of the region of M. 33 *Trianguli*, which was taken on November 14 last with the Dallmeyer $3\frac{1}{2}$-inch lens, and exposure of the plate during two hours and fifteen minutes. The circular photo-field shown is 11 degrees in diameter, and therefore contains 95 square degrees. In the centre of the circle are ruled four lines which enclose a rectangular space of four square degrees, and upon this area I counted 380 stars; and if we assume the stars to be equally numerous all over the field of 95 degrees there would be 9,025 stars upon it. The star-discs are round, sharply defined, and the faint ones small; the nebula is shown as a small patch or stain on the film, with very little detail and very few stars involved in it, and the plate all over is considerably fogged.

The next photograph, shown on the screen, was taken simultaneously with that just described, with the Cooke 5-inch lens. The circular field is 11 degrees in diameter, and represents 95 square degrees of the sky, which are coincident with those on the first photograph, and the four ruled lines also enclose identically the same area of four square degrees. Upon this area I counted 840 stars, and on the assumption of equal stars on equal

areas there would be 19,950 stars on the 95 degrees. The stars are round, sharply defined, and the faint ones small; a considerable extent of the spiral nebula is shown together with structural details and many of the faint and bright stars which are involved; the plate is free from any trace of fogging.

The third photograph, now shown, was taken with the 20-inch reflector simultaneously with the two already described, and it will be observed that the scale is large because of the larger instrument used. The area of the sky represented is only four square degrees, and is coincident with the areas that were shown on the first and second photographs, enclosed within the four lines referred to. I counted upon these four degrees 2,960 stars, and by the assumptions made and applied to the other two photographs there would be 70,300 stars on 95 square degrees. The nebula was described and illustrated before the Society at the meeting in December last, and I need not here repeat the description then given; suffice it to state that we have had projected upon the screen, and made evident to sight, demonstrations of the fact that on the reflector photograph the nebula is more extensively and clearly depicted, that it is at least two stellar magnitudes denser, and that far more of the structural details are shown than can be seen on the photographs taken with the portrait lenses.

The stars also are 3·52 times more numerous on the reflector photograph than on that by the 5-inch lens, and 7·78 times more numerous than on the photograph by the $3\frac{1}{2}$-inch lens.

The examination of the nebulosity and the counting of the stars was in each of these cases done upon the original negatives.

The second series of the tests consists of three photographs of the region of γ *Cassiopeiæ*, which were taken simultaneously on 1895 December 13, with an exposure of the plates during two hours and twelve minutes.

The first, now exhibited, was taken with the Dallmeyer $3\frac{1}{2}$-inch lens. The circular photo-field is 11 degrees in diameter, and therefore contains 95 square degrees. Upon an area of four square degrees with γ in the centre I counted 1,080 stars; and assuming, as in the other cases cited, that the stars are distributed equally over the plate, there would be 25,650 stars on 95 degrees. The photo-diameter of γ measures 18·7 minutes of arc, and the two fan-like nebulæ on the *north following* side are faintly shown but without structural details; the plate is fogged in irregular patches that might be easily mistaken for nebulosity.

The second photograph was taken with the Cooke 5-inch lens, and the circular field is 11 degrees in diameter, and represents 95 square degrees of the sky coincident with the first. Upon an area of four square degrees, corresponding with that on the first photograph, I counted 2,610 stars; and under the conditions already named there would be 61,987 stars on 95 degrees. The photo-diameter of γ measures 10·23 minutes of arc, and the two fan-like nebulæ, already referred to, are much more prominently

shown, together with some structural details and many of the involved stars; the plate is quite free from fogging effects and from spurious nebulosity.

The third photograph of this series is now shown; it was taken with the 20-inch reflector simultaneously and under precisely the same conditions as the first and second photographs just described; the photo-field is four degrees square, and upon this area I counted 17,100 stars; and again assuming equal stars on equal areas there would be 406,125 stars on 95 degrees. The photo-diameter of γ measures 16′·9 of arc; and the two fan-like nebulæ are very strongly and clearly shown with much structural detail; lights and shade; and lines and curves of stars involved making altogether not only a picture, but perpetuating each structural form, on a scale sufficiently large, for future scientific investigations and for the detection of any changes that must, some time or other, affect the character of the objects.

The stars are 6·55 times more numerous on the reflector photograph than on that by the 5-inch lens; and 15·93 times more numerous than by the $3\frac{1}{2}$-inch lens.

The examination of the nebulosity, and the counting of the stars was, in this and in the third series of tests, made upon the positive copies here exhibited, and not upon the negatives, for the reasons which will presently be given.

I may here state that the two fan-like nebulæ, which have been referred to, were faintly shown on a photograph of the region of γ which was taken with the 20-inch reflector on 1890 January 17, but at that time I thought they were due to stains on the film. The photo-plates were, in those days, less sensitive than they are now made.

The third series of the tests consists of three photographs of the region of the *Pleiades* which were taken simultaneously on 1896 February 4, with an exposure of the plates during two hours and fifty minutes.

The first, now exhibited, was taken with the Dallmeyer $3\frac{1}{2}$-inch lens; the circular photo-field is 11 degrees in diameter and contains 95 square degrees. Upon the area of four square degrees with η in the centre I counted 383 stars; and assuming, as in all the other cases cited, that the stars are distributed equally over the plate, there would be 9,100 stars on 95 degrees. The nebula round *Merope* is shown, but without any structural details and none of the other well-known nebulosities in the group can be seen. The disks of the bright stars are large and fuzzy, and the plate is much fogged.

The second photograph, now shown, was taken with the Cooke 5-inch lens; the circular field being 11 degrees in diameter and exhibiting 95 square degrees of the sky coincident with the first. Upon the area of four square degrees corresponding with that on the first photograph, I counted 953 stars; and upon 95 degrees there would be 22,600 stars. The *Merope* nebula is well shown and with some detail; so also are the other prominent

nebulosities in the group, and the plate is free from halation effects and fog. There is no indication of other nebulosity on other parts of the 95 degrees.

The last photograph of the series was taken with the 20-inch reflector, simultaneously with the two just described, and it exhibits four square degrees upon which I counted 3,470 stars; and upon 95 degrees there would be 82,400 stars. The stars are slightly elongated, but all the now known nebulosities are densely and brightly shown; crowded with details and free from halation and fog.

The stars are 3·64 times more numerous on the reflector photograph than on that by the 5-inch lens; and 9·06 times more numerous than by the $3\frac{1}{2}$-inch lens.

In all the experiments which I have made, the nebulosity, shown on the plates taken with the reflector, is denser than that by the portrait lenses in the ratio, approximately, of the relative number of faint stars shown on the plates which have been simultaneously exposed in the three instruments; and the illustrations which have been exhibited are proofs of the justness of this inference.

The results of the experiments herein described prove conclusively the greater efficiency of the reflector form of instrument over the portrait-lens or refractor form, for certain work in celestial photography; and it is not probable that refractors having shorter foci to apertures than 1 to 2·74 and 1 to 4·8, such as were used in these experiments, can, with greater perfection, be made of 20 inches aperture and 98 inches focus, so as to give better comparative results than those herein recorded.

These experiments also point to a practical limit of aperture to focus in the construction of instruments for celestial photography; and that the limit lies very nearly as 1 to 5; for I have consistently found a deterioration in the stellar images below aperture of 1 to 4; and apertures above 1 to 6 are slower in photographic effect.

Much misapprehension exists concerning the stellar image as seen in a telescope, and the photo-image as seen on a plate; it arises from the confusion of the ideas of points and surfaces of light. The telescopic image may be a point, but the photo-image is a measurable surface of chemical effect, spread on the sensitised film; and partakes of the character of nebulæ of small dimensions. As such, they must be subject to a rule which is not applicable to the visual telescopic image.

It follows from this that it is not possible, as it has been asserted, that a photographic instrument of the portrait lens form can imprint on the film images of nebulæ that are fainter than the faintest star-images imprinted at the same time and under exactly similar conditions.

The deductions which I have so far made are based upon the performances of lenses of a high character; but the ordinary portrait lenses are largely affected with imperfections producing

what are known as ghosts, and flares, in addition to fogging; therefore those who engage in stellar photography should first satisfy themselves that their lenses are free from these defects, and, above all, be careful that their plates are efficiently backed with some substance that will prevent reflections of the light from bright stars causing nebulous circles of halation on the film.

As illustrations of some of these defects, and also in further confirmation of the reliability of the results already given respecting the reflector and the portrait lenses, I will refer to some photo-positives on glass which are to be seen in the Library of the Society. They were taken by Professor Barnard at the Lick Observatory with what is described as the "Willard" lens, which has an aperture of 6 inches, and focus of 30·82 inches; the ratio of aperture to focus is therefore 1 to 5·14.

I shall refer only to three of these photographs, which are typical of the whole series. (1) The region of γ *Cassiopeiæ* was photographed on 1894 February 2, with an exposure of the plate during three hours; the photo-field is about $13\frac{1}{2}$ degrees in diameter, of which about 11 degrees are fairly free from distortion. γ is seen in the middle of a halation circle of about 28 minutes of arc in diameter; and the two fan-like nebulæ are shown, but with little if any structural details and only a few of the involved stars are visible. I counted 1,300 stars on 2° by 2° with γ in the centre, and this area coincides with the respective areas of four square degrees which have already been described and illustrated, when referring to the photographs taken with the two lenses and the reflector.

The results of the comparison of this photograph with the others are as follows :—

$3\frac{1}{2}$-inch lens; focus 2·74; exposure 132 minutes; 1,080 stars on 2° by 2°; the two nebulæ are visible but without structural details.

5-inch lens; focus 4·8; exposure 132 minutes; 2,610 stars on 2° by 2°; the nebulæ are well shown and some of the involved stars also.

6-inch Willard lens; focus 5·4; exposure 180 minutes; 1,300 stars on 2° by 2°; the two nebulæ are shown with some of the stars involved in them, but with less density and clearness than by the 5-inch lens.

20-inch reflector; focus 4·9; exposure 132 minutes; 17,100 stars on 2° by 2°; the two nebulæ are brilliantly shown, with structural details, and many stars both bright and faint involved in the nebulosity.

The other two photographs to which I shall refer are of the region of the *Pleiades*, taken with the Willard lens: one with an exposure of 4 hours on 1893 December 1, and the other with an exposure of $10\frac{1}{2}$ hours on 1893 December 6–8.

That with 4 hours' exposure shows the *Merope* nebula, but without any structural details; and there is a part of the projecting nebula from *Electra*; but all the rest of the nebulosity in

the group is lost in the halation circles which surround each of the bright stars, and none of the distant nebulosity is shown. I counted on four square degrees, with η in the centre, 825 stars as against 953 on the 5-inch lens plate, and 3,470 stars on the reflector plate; the exposure of the two last named being $2^h\ 50^m$ against 4^h with the Willard lens.

The plate exposed during $10\frac{1}{2}$ hours shows only 1,385 stars on $2°$ by $2°$, whilst the same area on the 4^h plate has 1,259 stars; therefore the difference in photo-effect upon the two plates is less than that of one stellar magnitude. This fact throws some doubt upon the reality of the distant nebulosity which is shown on the plate; and when we consider that the whole patch, that covers the group of the *Pleiades*, is due to halation, and not to nebulosity, the doubt is further strengthened.

The star images on this photograph are double and overlap; the exposures of the plate, therefore, were effectively of less than $10\frac{1}{2}$ hours' duration.

Photograph of the "Owl" Nebula M 97 and of the Nebula H V 46 *Ursæ Majoris.* By Isaac Roberts, D.Sc., F.R.S.

The photograph of the nebula M 97, R.A. $11^h\ 8^m\ 42^s$, Decl. 55° 36′ (epoch 1895), and of H V 46, R.A. $11^h\ 5^m\ 22^s$, Decl. 56° 15′ north, was taken with the 20-inch reflector on 1895 April 20, with an exposure of the plate during four hours, and the copy now presented is enlarged to the scale of 1 millimetre to 15 seconds of arc.

The nebula M 97 is N.G.C. No. 3587, G.C. No. 2343, *h* 838. Rosse, *Obs. of Neb. and Cl. of Stars*, p. 93, and *Phil. Trans.* 1850, Pl. XXXVII. fig. 11.

Sir J. Herschel (G.C. 2343) describes it as a very remarkable object, a planetary nebula, very bright, very large, round, very, very gradually, then very suddenly brighter in the middle, $19^s{\cdot}0$ in diameter. It is figured in the *Phil. Trans.* 1833 as a circle, stippled without any details.

Lord Rosse (cited above), in 1850, figured the nebula as a circle filled in with details somewhat resembling the face of an owl, with hair-like projections round the margin; and between the years 1848 and 1874 records the particulars of forty-five observations which were made. In some of them both he and Dr. Robinson saw a faint star to the right of the central star, and suspected the existence of one or two other very faint stars, as well as a spiral shape, but he does not refer to the hair-like surroundings of the nebula.

The photograph, now projected on the screen, shows the nebula as an ellipse with the major axis in *north following* to *south preceding* direction: it measures about 203 seconds of arc